# MATH 110 NOTES

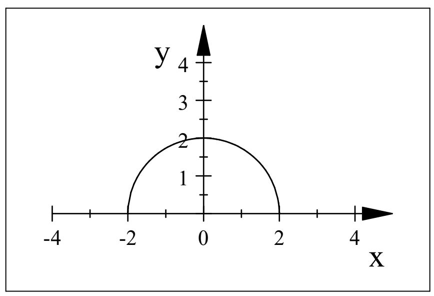
## NOTE 1:

### 1.1: Domain and Range of some functions

| Function | Domain | Range |
|---|---|---|
| $\sqrt{a^2 - x^2}$ | $[-a, a]$ | $[0, a]$ |
| $\sqrt{x^2 - a^2}$ | $(-\infty, -a] \cup [a, \infty)$ | $[0, \infty)$ |
| $\sqrt{x^2 + a^2}$ | $\mathbb{R}$ | $[a, \infty)$ |

**Examples**:

1. $f(x) = \sqrt{4 - x^2}$




Domain $(f) = [-2, 2]$, Range $(f) = [0, 2]$.

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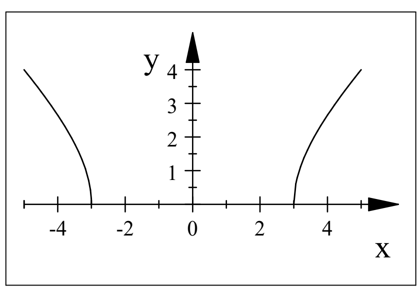
2. $f(x) = \sqrt{x^2 - 9}$




Domain $(f) = (-\infty, -3] \cup [3, \infty)$, Range $(f) = [0, \infty)$.

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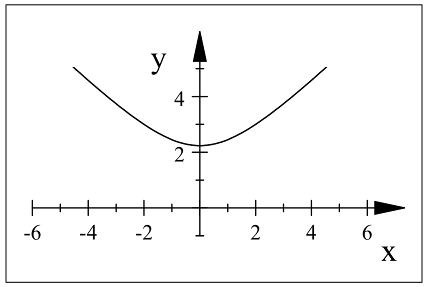
3. $f(x) = \sqrt{x^2 + 5}$




Domain $(f) = \mathbb{R}$, Range $(f) = [\sqrt{5}, \infty)$.

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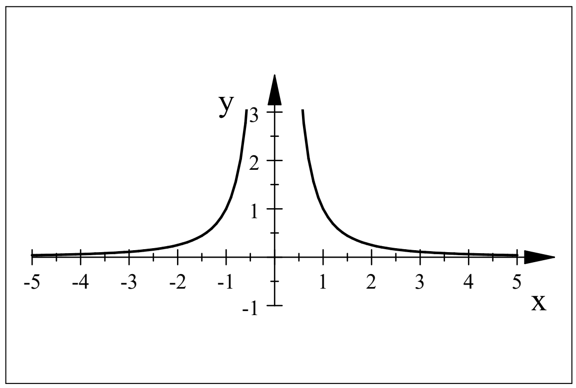
**Example**: The function whose graph is given is




**a**) increasing on $(-\infty, 0)$.

**b**) increasing on $(0, \infty)$.

**c**) decreasing on $(-\infty, 0)$

**d**) decreasing on $\mathbb{R}$.

solution: a

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# NOTE 2:

**1.2**: **Mathematical models.**

**Rational Functions**: $R(x) = \dfrac{f(x)}{g(x)}$ is rational function if $f(x)$ and $g(x)$ are polynomials.

The domain of $R(x)$ is $\mathbb{R} - \{$zeros of the denominator $g(x)\}$.

**Examples**:

1. $h(x) = \dfrac{3x - 1}{x^2 + 5x - 6}$, Domain $(h) = \mathbb{R} - \{-6, 1\}$.
2. $f(x) = \dfrac{5x^4 - 3}{x^2 + 4}$, Domain $(f) = \mathbb{R}$

**Remarks**:

1. The range of a function is required for the known functions (all functions in 1.2). It is also required to be obtained from a given graph of any function.
2. The domain of a function is required to be obtained from the formula of a function as well as from a given graph.

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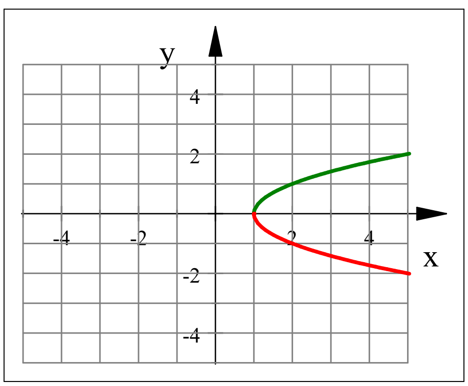
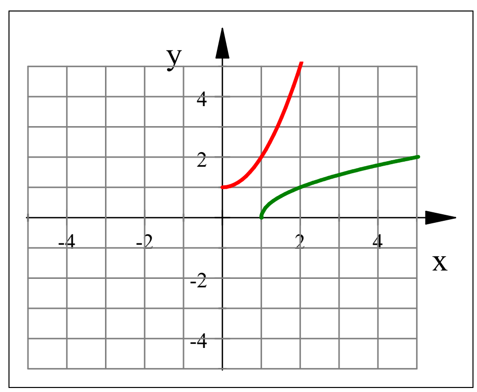
# NOTE 3:

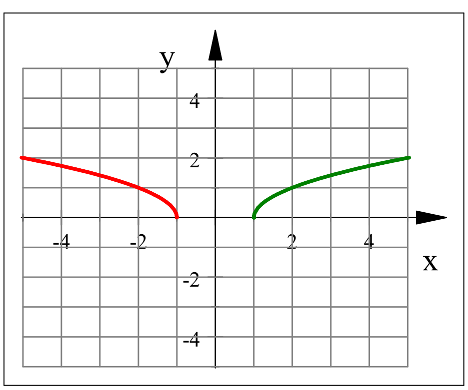
## 1.5: Inverse Functions and Logarithms

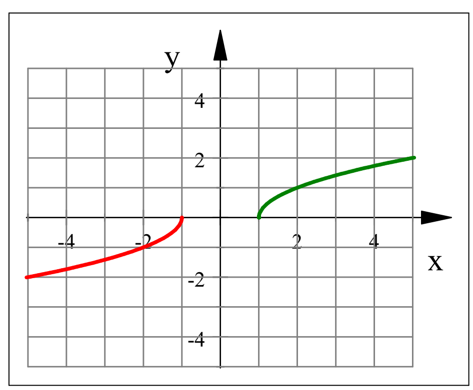
**Example**:

The figure which represent a graph of a function and it's inverse at the same coordinates axes is

a) 


b) 


c) 


d) 


solution: b

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# NOTE 4:
**2.2**: **The limit of a function**

$$\lim_{x\to a}\frac{1}{(x-a)^n} = \begin{cases} \infty & n \text{ even} \\ n \text{ odd} \begin{cases} \lim_{x\to a^+}\frac{1}{(x-a)^n} & = \infty \\ \lim_{x\to a^-}\frac{1}{(x-a)^n} & = -\infty \end{cases} \end{cases}$$

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# NOTE 5:
**2.5**: **Continuity**.

**Examples**:

**1**. The function $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$ is

**a**) discontinuous at $x = 0$.

**b**) continuous on $\mathbb{R}$.

**c**) continuous only on $(-\infty, 0) \cup (0, \infty)$.

**d**) continuous only on $(0, \infty)$.

solution: b

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**2**. The function $f(x) = \begin{cases} x-1 & \text{if } x < -1 \\ 2 & \text{if } -1 \leq x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases}$ is discontinuous at

**a**) $\{-1, 1\}$

**b**) $\{1\}$

**c**) $\{-1\}$

**d**) $[-1, 1]$

solution: c

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**Examples on continuity from one side**:

1. The function $f(x) = \sqrt{3-x}$ is continuous from the right at $x = 3$.

 **a**) True

 **b**) False

 solution: b

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2. The function $f(x) = \sqrt{x-3}$ is

 **a**) continuous at $x = 3$.

 **b**) continuous from the right at $x = 3$.

 **c**) continuous from the left at $x = 3$.

 **d**) continuous at $x = 0$.

 solution: b

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3. The function $f(x) = \begin{cases} x+1 & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases}$ is

 **a**) continuous from the right at $x = 2$.

 **b**) continuous from the left at $x = 2$.

 **c**) continuous on $\mathbb{R}$.

 **d**) continuous at $x = 2$.

 solution: a

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# NOTE 6:

**2.6**: **Limits at infinity**.

**Remarks**:

1. $\lim_{x\to\pm\infty}(a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + ax + a_0) = \lim_{x\to\pm\infty}(a_n x^n)$

2. $\lim_{x\to\pm\infty} x^n == \begin{cases} \infty & n \text{ even} \\ n \text{ odd} \begin{cases} \lim_{x\to+\infty} x^n & = \infty \\ \lim_{x\to-\infty} x^n & = -\infty \end{cases} \end{cases}$

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# NOTE 7:

**3.3: Derivatives of trigonometric functions.**

**Remarks:**

**a)** $\lim_{x\to 0} \frac{\sin(nx)}{mx} = \frac{n}{m}$

**b)** $\lim_{x\to 0} \frac{mx}{\sin(nx)} = \frac{m}{n}$

**c)** $\lim_{x\to 0} \frac{\tan(nx)}{mx} = \frac{n}{m}$

**d)** $\lim_{x\to 0} \frac{mx}{\tan(nx)} = \frac{m}{n}$

**e)** $\lim_{x\to 0} \frac{\tan(nx)}{\sin(mx)} = \frac{n}{m}$

**f)** $\lim_{x\to 0} \frac{\sin(mx)}{\tan(nx)} = \frac{m}{n}$

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